ON $\pi - \pi$ THEOREM FOR MANIFOLD
PAIRS WITH BOUNDARIES

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ABSTRACT. Surgery obstruction of a normal map to a simple Poincare pair (X, Y) lies in the relative surgery obstruction group $L_*(\pi_1(Y) \to \pi_1(X))$. A well known result of Wall, the so called π-π theorem, states that in higher dimensions a normal map of a manifold with boundary to a simple Poincare pair with $\pi_1(X) \cong \pi_1(Y)$ is normally bordant to a simple homotopy equivalence of pairs. In order to study normal maps to a manifold with a submanifold, Wall introduced surgery obstruction group for manifold pairs LP_* and splitting obstruction groups LS_*. In the present paper we formulate and prove for manifold pairs with boundaries the results which are similar to the π-π theorem. We give direct geometric proofs, which are based on the original statements of Wall's results and apply obtained results to investigate surgery on filtered manifolds.

1. Introduction.

The surgery obstruction groups $L_*(\pi)$ were introduced by Wall in his fundamental paper [7] . Let $(f, b) : M \to X$ be a normal map from a closed manifold M^n to a simple Poincaré complex X of formal dimension n where $b : \nu_M \to \xi$ is a bundle map which covers $f : M \to X$. Then an obstruction $\theta(b, f) \in L_n(\pi_1(X))$ for the existence of a simple homotopy equivalence in the class of normal bordism of the map (f, b) is defined.

Indeed, Wall defines L_*-groups and describes surgery theory for the case of manifold n-ads. For example, an obstruction to surgery on the manifold pair lies in the relative surgery obstruction group $L_*(\pi_1(Y) \to \pi_1(X))$. Hence, if the map $\pi_1(Y) \to \pi_1(X)$ is an isomorphism, then in high dimensions a normal map from a manifold with boundary to the simple Poincare pair is normally bordant to a simple homotopy equivalence of pairs. In [7, §4] Wall gives direct geometric proof of this "important special case".

The surgery obstruction groups and natural maps do not depend on the category of manifolds (see [6] and [7]). In the present paper we shall work in the category of topological manifolds. All results of this paper are transferred to $Diff$ or PL-manifolds.

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Let (X^n, Y^{n-q}, ξ) be a codimension q manifold pair [6, §7.2]. Let

$$
(1) \qquad F = \begin{pmatrix} \pi_1(S(\xi)) & \longrightarrow & \pi_1(X \setminus Y) \\ \downarrow & & \downarrow \\ \pi_1(Y) & \longrightarrow & \pi_1(X) \end{pmatrix},
$$

be the pushout square of fundamental groups with orientations where $S(\xi)$ is the boundary of the tubular neighborhood of Y in X.

The obstruction groups $LP_*(F)$ to surgery on the manifold pair (to obtain s-triangulation of the pair) are defined (see [7, §17E] and [6, §7.2]). A simple homotopy equivalence $f : M \to X$ splits along a submanifold Y if it is homotopy equivalent to a map g which is an s-triangulation of the manifold pair (X, Y, ξ). The splitting obstruction groups $LS_*(F)$ are defined in [6] and [7].

Let

$$
(2) \qquad (Y, \partial Y) \subset (X, \partial X)
$$

be a codimension q manifold pair with boundaries (see [6, §7]). A normal map

$$
(3) \qquad (f, \partial f) : (M, \partial M) \to (X, \partial X)
$$

is an s-triangulation of the manifold pair (X, Y) with boundaries $(\partial X, \partial Y)$ if the maps $f : M \to X$ and $\partial f : \partial M \to \partial X$ are s-triangulations of the pairs (X, Y) and $(\partial X, \partial Y)$, respectively.

In rel_∂ case we consider normal maps (3) which are already split on the boundary. The surgery theory for this case was developed in [6] and [7]. In the rel_∂-case the obstructions to surgery on manifold pairs relatively the boundary lie in the group $LP_*(F)$, and similarly for rel_∂ splitting obstruction (see [6, pages 584–587]).

In the present paper we consider surgery on manifold pairs with boundaries without fixing of maps on the boundary. To study surgery on filtered manifolds (see [1], [5], and [8]) we need to know the geometric properties of surgery on manifold pairs with boundaries. This is the special case of splitting theory for manifolds n-ads mentioned by Wall [7, page 136]. We give the exact statement and proof of $\pi - \pi$-theorem for various maps to a manifold pair with boundary. Then we apply obtained results to investigation of surgery on filtered manifolds.

A manifold pair (2) with boundary defines a pair of closed manifolds $\partial Y \subset \partial X$ with a pushout square

$$
(4) \qquad \Psi = \begin{pmatrix} \pi_1(S(\partial \xi) & \longrightarrow & \pi_1(\partial X \setminus \partial Y) \\ \downarrow & & \downarrow \\ \pi_1(\partial Y)) & \longrightarrow & \pi_1(\partial X) \end{pmatrix}
$$

of fundamental groups for the splitting problem. Here $\partial \xi$ is the restriction of ξ on the boundary ∂Y.

A natural inclusion $\delta : \partial X \to X$ induces a map of $\Delta : \Psi \to F$ of squares of fundamental groups.

Now we can formulate the main result of the paper.

Theorem. *Let $(Y^{n-q}, \partial Y) \subset (X^n, \partial X)$ be a codimension q manifold pair with boundary with $n - q \geq 6$. Let the map Δ be an isomorphism of the squares of fundamental groups. Under these conditions we have the following results.*

A) Any normal map (3) is normally bordant to an s-triangulation of $(X, \partial X)$ which is split along $(Y, \partial Y) \subset (X, \partial X)$.

B) Any simple homotopy equivalence (3) of pairs is concordant to a simple homotopy equivalence to $(X, \partial X)$ which is split along $(Y, \partial Y) \subset (X, \partial X)$.

C) Let a normal map (3) define a simple homotopy equivalence of pairs

$$f|_{(N, \partial N)} : (N, \partial N) \to (Y, \partial Y)$$

where $N = f^{-1}(Y)$, $\partial N = f^{-1}(\partial Y)$ are transversal preimages. Then $(f, \partial f)$ is normally bordant to an s-triangulation of $(X, \partial X)$ which is split along $(Y, \partial Y) \subset (X, \partial X)$. Moreover, there exists a transversal to $Y \times I$ bordism

$$F = (h; g, f_0, f_1) : (W; V, M, M') \to (X \times I; \partial X \times I, X \times \{0\}, X \times \{1\})$$

for which the restriction $h_{h^{-1}(Y \times I)}$ is

$$(f|_{(N, \partial N)}) \times \mathrm{Id} : (N, \partial N) \times I \to (Y, \partial Y) \times I$$

In section 2 we give necessary preliminary material. In section 3 we prove the theorem and apply our results to surgery on filtered manifolds.

2. Preliminaries.

We shall consider a case of topological manifolds and follow notations from [6, §7.2]. Let (X, Y, ξ) be a codimension q manifold pair in the sense of Ranicki (see [6, §7.2]) i. e. a locally flat closed submanifold Y is given with a normal fibration

$$\xi = \xi_{Y \subset X} : Y \to \widetilde{BTOP}(q)$$

with the associated (D^q, S^{q-1}) fibration

$$(D^q, S^{q-1}) \to (E(\xi), S(\xi)) \to Y$$

and we have a decomposition of the closed manifold

$$(5) \qquad \qquad X = E(\xi) \cup_{S(\xi)} \overline{X \setminus E(\xi)}.$$

A topological normal map [6, §7.2]

$$((f, b), (g, c)) : (M, N) \to (X, Y)$$

to the manifold pair $(X, Y, \xi))$ is represented by a normal map (f, b) to the manifold X which is transversal to Y with $N = f^{-1}(Y)$, and (M, N) is a topological manifold pair with a normal fibration

$$\nu : N \xrightarrow{f|_N} Y \xrightarrow{\xi} \widetilde{BTOP}(q).$$

Additionaly the following conditions are satisfied:

(i) the restriction

$$(f,b)|_N = (g,c) : N \to Y$$

is a normal map;

(ii) the restriction

$$(f,b)|_P = (h,d) : (P,S(\nu)) \to (Z,S(\xi))$$

is a normal map to the pair $(Z,S(\xi))$, where

$$P = \overline{M \setminus E(\nu)}, \quad Z = \overline{X \setminus E(\xi)};$$

(iii) the restriction

$$(h,d)|_{S(\nu)} : S(\nu) \to S(\xi)$$

coincides with the induced map

$$(g,c)^! : S(\nu) \to S(\xi),$$

and $(f,b) = (g,c)^! \cup (h,d)$.

The normal maps to (X,Y,ξ) are called t-triangulations of the manifold pair (X,Y). Note that the set of concordance classes of t-triangulations of the pair (X,Y,ξ) coincides with the set of t-triangulations of the manifold X [6, Proposition 7.2.3].

An s-triangulation of a manifold pair (X,Y,ξ) in topological category [6, p. 571] is a t-triangulation of this pair for which the maps

(6) $$f : M \to X, \ g : N \to Y, \text{ and } (P,S(\nu)) \to (Z,S(\xi))$$

are simple homotopy equivalences (s-triangulations).

For a codimension q manifold pair with boundaries (2) [6, p. 585] we have a normal fibration $(\xi,\partial\xi)$ over the pair $(Y,\partial Y)$ and, similarly to (5), a decomposition

(7) $$(X,\partial X) = (E(\xi) \cup_{S(\xi)} Z, E(\partial\xi) \cup_{S(\partial\xi)} \partial_+ Z)$$

where $(Z;\partial_+ Z, S(\xi); S(\partial\xi))$ is a manifold triad. Note here that $\partial_+ Z = \overline{\partial X \setminus E(\partial\xi)}$.

A topological normal map (4) of manifold pairs with boundaries provides a normal fibration $(\nu,\partial\nu)$ over the pair $(N,\partial N)$ where $(N,\partial N) = (f^{-1}(Y),(\partial f)^{-1}(\partial Y))$ [6, p. 570]. We have the following decomposition

(8) $$(M,\partial M) = (E(\nu) \cup_{S(\nu)} P, E(\partial\nu) \cup_{S(\partial\nu)} \partial_+ P)$$

where $(P;\partial_+ P, S(\nu); S(\partial\nu))$ is a manifold triad. Recall, that two s-triangulations of the pair $(X,\partial X)$

$$(f_i,\partial f_i) : (M_i,\partial M_i) \to (X,\partial X), \ i = 0,1$$

are concordant (see [7, §10] and [6, §7.1]) if there exists a simple homotopy equivalence of 4-ads

(9) $$(h;g,f_0,f_1) : (W;V,M_0,M_1) \to (X \times I; \partial X \times I, X \times \{0\}, X \times \{1\})$$

with

$$\partial V = \partial M_0 \cup \partial M_1.$$

3. Proof and Corollary.

Consider the case A) of the theorem. A restriction of the map

$$(f, \partial f) : (M, \partial M) \to (X, \partial X)$$

gives a normal map of pairs

(10) $$\phi = f|_{S(\nu)} : (S(\nu), S(\partial \nu)) \to (S(\xi), S(\partial \xi)).$$

The isomorphism Δ provides an isomorphism $\pi_1(S(\partial \xi)) \to \pi_1(S(\xi))$. Hence the normal map $\phi = f|_{S(\nu)}$ (10) of the pairs satisfies the conditions of $\pi - \pi$ theorem of Wall [7, §4] and it is normally bordant to a simple homotopy equivalence of pairs. By [3, page 45] we can extend this bordism to obtain a normal bordism with the bottom map $(f, \partial f)$ and with the top a normal map

$$(f', \partial f') : (M', \partial M') \to (X, \partial X)$$

with the properties similar to the map f and for which the restriction

$$\phi' = f'|_{S(\nu')} : (S(\nu'), S(\partial \nu')) \to (S(\xi), S(\partial \xi)).$$

is a simple homotopy equivalence of pairs. To avoid complicated notations we can suppose that the normal map (4) has the restriction ϕ (10) which is a simple homotopy equivalence of pairs.

Now the restriction of the map $(f, \partial f)$ gives a normal map of triads

$$\psi = f|_{E(\nu)} : (E(\nu); E(\partial \nu), S(\nu); S(\partial \nu)) \to (E(\xi); E(\partial \xi), S(\xi); S(\partial \xi)).$$

The restriction
$$\psi|_{S(\nu)} = \phi : (S(\nu), S(\partial \nu)) \to (S(\xi), S(\partial \xi))$$

is the simple homotopy equivalence of pairs, and the isomorphism Δ provides the isomorphism $\pi_1(E(\partial \xi)) \to \pi_1(E(\xi))$. Hence the normal map ψ satisfies the conditions of $\pi - \pi$ theorem for the triad [7, Theorem 3.3] and it is normally bordant to a simple homotopy equivalence of triads by bordism $\Phi : V \to E(\xi) \times I$ relatively $(S(\nu), S(\partial \nu))$ with the bottom map $\psi : V_0 = E(\nu) \to E(\xi)$. The restriction of (Φ, V) to the $S(\nu)$ is a trivial bordism

$$\phi_t : Q = S(\nu) \times I \to S(\xi) \times I, \ \phi_t(x, t) = (\phi(x), t), \ t \in I = [0, 1].$$

We can attach the bordism V to the manifold M identifying V_0 with $E(\nu)$ to obtain a space $\Lambda = V \cup_{E(\nu)} M$. Since the restriction of Φ to the bottom $V_0 = E(\nu)$ coincides with the map $\psi = f|_{E(\nu)}$ we obtain the map

$$F = \Phi \cup_\psi f : \Lambda = V \cup_{E(\nu)} M \to (E(\xi) \times I) \cup_{E(\xi) \times \{0\}} X.$$

In a similar way, the restriction of the map $(f, \partial f)$ gives a normal map of triads

$$\alpha = f|_P : (P; \partial_+ P, S(\nu); S(\partial \nu)) \to (Z; \partial_+ Z, S(\xi); S(\partial \xi)),$$

for which the restriction

$$\alpha|_{S(\nu)} = \phi : (S(\nu), S(\partial\nu)) \to (S(\xi), S(\partial\xi))$$

is a simple homotopy equivalence of pairs. The isomorphism Δ provides an isomorphism $\pi_1(\partial_+ Z) \to \pi_1(Z)$. Hence the normal map α is normally bordant to a simple homotopy equivalence of triads by bordism $G : W \to Z \times I$ relatively $(S(\nu), S(\partial\nu))$ with the bottom map

$$G|_{W_0} = \alpha = f|_P : W_0 = P \to Z.$$

We can attach W to the space Λ to obtain a space $\Lambda \cup_{Q \cup P} W$. The restriction $G|_{W_0}$ coinsides with the map $\alpha = f|_P$ and bordism maps F and G coincides on Q. Thus we obtain a bordism

(11)
$$\Omega : \Lambda \cup_{Q \cup P} W \to X \times I$$

where the map Ω extends the maps F and G. By our construction, on the top of the bordism (11) we obtain the map

$$(f', \partial f') : (M', \partial M') \to (X, \partial X)$$

for which the restrictions give the simple homotopy equivalences of triads

$$(E(\nu'); E(\partial\nu'), S(\nu'); S(\partial\nu') \to (E(\xi); E(\partial\xi), S(\xi); S(\partial\xi))$$

and

$$(P'; \partial_+ P', S(\nu'); S(\partial\nu')) \to (Z; \partial_+ Z, S(\xi); S(\partial\xi)),$$

where

$$(M', \partial M') = (E(\nu') \cup_{S(\nu')} P', E(\partial\nu') \cup_{S(\partial\nu')} P'_+).$$

To finish the proof of A) we must verify only that the constituent maps $f' : M' \to X$ and $\partial f' : \partial M' \to \partial X$ are simple homotopy equivalences.

The space M' is union of two parts

$$P' \cup_{S(\nu')} E(\nu')$$

with the intersection $S(\nu')$. The restrictions of f on these two parts and on the intersection are simple homotopy equivalences. Hence by [2, Theorem 23.1] the map $f' : M' \to X$ is simple homotopy equivalence. For the map $\partial f'$ the situation is similar since

$$\partial M' = \partial_+ P' \cup_{S'(\partial\nu)} E'(\partial\nu).$$

The case A) is proved.

Now consider the case B). The map $(f, \partial f)$ (4) is a simple homotopy equivalence of pairs. Since the map Δ is an isomorphism then by A) the map $(f, \partial f)$ is normally bordant to a map

$$(f', \partial f') : (M', \partial M') \to (X, \partial X)$$

which is split along $(Y, \partial Y) \subset (X, \partial X)$. Thus we have a normal bordism

$$\Phi : W \to X \times I, \ \partial W = W_0 \cup W_1 \cup V$$

where
$$W_0 = M, W_1 = M', V = \Phi^{-1}(\partial X \times I).$$

The bordism (W, Φ) gives a normal map of manifold triads

(12) $(W; V, W_0 \cup W_1; \partial W_0 \cup \partial W_1) \to (X \times I; \partial X \times I, X \times \{0, 1\}; \partial X \times \{0, 1\})$

which we shall denote by Φ, too.

The restriction of the normal map Φ on $W_0 \cup W_1$ is the simple homotopy equivalence of pairs

$$(W_0 \cup W_1, \partial W_0 \cup \partial W_1) \to (X \times \{0, 1\}; \partial X \times \{0, 1\}).$$

The fundamental group of the triad

$$(X \times I; \partial X \times I, X \times \{0, 1\}; \partial X \times \{0, 1\})$$

is equal to

(13) $F_I = \begin{pmatrix} \pi_1(\partial X \times \{0, 1\})) & \longrightarrow & \pi_1(\partial X \times I) \\ \downarrow & & \downarrow \\ \pi_1(X \times \{0, 1\})) & \longrightarrow & \pi_1(X \times I) \end{pmatrix}.$

The isomorphism Δ provides the isomorphism $\pi_1(\partial X) \to \pi_1(X)$ and, hence, the vertical maps in the square (13) are isomorphisms of grouppoids. Hence the normal map of triads (12) satisfies the conditions of $\pi - \pi$ theorem for triads relatively

$$\partial_2(X \times I) = ((\partial X \times \{0, 1\}) \subset (X \times \{0, 1\})).$$

Thus the map Φ (12) of triads is normally bordant relatively $\partial_2(X \times I)$ to a simple homotopy equivalence of triads

$\Phi' : (W'; V', W_0 \cup W_1; \partial W_0 \cup \partial W_1) \to (X \times I; \partial X \times I, X \times \{0, 1\}; \partial X \times \{0, 1\})$

where
$$\Phi'|_{W_0} = f, \ \Phi'|_{W_1} = f', \ \Phi'|_{\partial W_0} = \partial f, \ \Phi'|_{\partial W_1} = \partial f'.$$

By our construction the map $(f', \partial f')$ is splitted along the pair $(Y, \partial Y) \subset (X, \partial X)$ and by (9) the map Φ' gives the concordance between $(f, \partial f)$ and $(f', \partial f')$. The part B) of the theorem is proved.

In the case C) the map f is transversal to $(Y, \partial Y)$ and its restriction is a simple homotopy equivalence $(N, \partial N) \to (Y, \partial Y)$. Hence $f|_N$ induces a simple homotopy equivalence of tubular neighborhoods with boundaries (see [7, page 8] and [6, page 579]). We obtain a simple homotopy equivalence of triads

$$\psi : (E(\nu); E(\partial \nu), S(\nu); S(\partial \nu)) \to (E(\xi); E(\partial \xi), S(\xi); S(\partial \xi))$$

for which the restriction
$$\psi|_{(N, \partial N)} = f|_{(N, \partial N)}$$

is the simple homotopy equivalence of pairs. Now the result follows from consideration of the map α of triads from A) by the same arguments. The theorem is proved. $\square$

Now we apply obtained results to surgery on filtered manifolds (see [1], [4], [5], and [8]). At first we recall necessary definitins.

Let $Z^{n-q-q'} \subset Y^{n-q} \subset X^n$ be a triple of closed topological manifolds (see [4], [5], and [6]). We have the following topological normal bundles: ξ for the submanifold Y in X, η for the submanifold Z in Y, and ν for the submanifold Z in X. Let $(E(\xi), S(\xi))$, $(E(\eta), S(\eta))$, and $(E(\nu), S(\nu))$, respectively, be the spaces with boundaries of associated (D^*, S^{*-1}) fibrations. We identify the space $E(\nu)$ with the space $E'(\xi)$ of the restriction $\xi|_{E(\eta)}$ in such a way that

$$(14) \qquad\qquad S(\nu) = E''(\xi) \cup S'(\xi)$$

where $E''(\xi)$ is the space of the restriction $\xi|_{S(\eta)}$ and $S'(\xi)$ is the restriction of $S(\xi)$ on $E(\eta)$ (see [1] and [5]).

Let

$$(15) \qquad (X_k, \partial X_k) \subset (X_{k-1}, \partial X_{k-1}) \subset \cdots \subset (X_0, \partial X_0) = (X, \partial X)$$

be a filtration of a compact manifold $(X, \partial X)$ by manofolds with boundaries (see [1], [5], and [8]). From now we shall assume that the dimension $\dim X_k \geq 6$.

The filtration (15) defines the filtration

$$(16) \qquad\qquad \partial X_k \subset \partial X_{k-1} \subset \cdots \subset \partial X_0 = \partial X$$

of ∂X by closed manifolds. Recall that any triple of manifolds from (15) and (16) satisfy properties that are similiar to (14) on the corresponding normal bundles. Additionaly, for every pair of manifolds with boundaries from (15) we have a decomposition that is similiar to (7). The filtration (15) defines a stratified manifold with boundary $(\mathcal{X}, \partial \mathcal{X})$ (see [1], [5], and [8]).

Any topological normal map (3) defines the topological normal map to the filtration (15) (see [5] and [8]). Let M_i be the transversal preimage of the submanifold X_i. A topological normal map (3) is an s-triangulation of the filtration (15) if constituent normal maps

$$f|_{(M_j, M_i)} : (M_j, M_i) \to (X_j, X_i), \ 0 \leq j \leq i \leq k$$

are s-triangulations of the manifold pairs with boundaries (X_j, X_i). The stratified Browder-Quinn surgery obstruction groups $L_n^{BQ}(\mathcal{X}, \partial \mathcal{X})$ are defined [1].

For $(1 \leq i \leq k)$, let F_i be the square in the splitting problem for the manifol pair $X_i \subset X_{i-1}$, and Ψ_i be the similiar square for the closed manifold pair $\partial X_i \subset \partial X_{i-1}$. The natural inclusions of boundaries induce the maps

$$\Delta_i : \Psi_i \to F_i$$

for $1 \leq i \leq k$.

Corollary. *Let all the maps Δ_i $(1 \leq i \leq k)$ be isomorphisms. Then every normal map to the filtered space $\mathcal{X}$ is normally bordant to an s-triangulation of $\mathcal{X}$ and, hence, the group $L_n^{BQ}(\mathcal{X}, \partial \mathcal{X})$ is trivial.*

Proof. Denote by f_{k-1} the restriction of the normal map f on the manifold M_{k-1}. By item A) of the theorem the map f_{k-1} is normally bordant to an s-triangulation

g_{k-1} of the pair (X_{k-1}, X_k). By [3] we can extend this bordism to obtain a bordism $F : W \to X \times I$ with a top normal map $(g, \partial g)$ to $(X, \partial X)$ whose restriction on M_{k-1} is the s-triangulation g_{k-1}.

Consider the resriction $(g, \partial g)|_{M_{k-2}}$ for which the the restriction on M_{k-1} is the s-triangulation g_{k-1}. By item C) of the theorem the map $(g, \partial g)|_{M_{k-2}}$ is normally bordant to a map g_{k-2} which is an s-triangulation of the pair (X_{k-2}, X_{k-1}) and

$$g_{k-2}\big|_{M_{k-1}} = g_{k-1}.$$

Now, by applying item C) $(k - 1)$ time, we obtain a map

$$(g_0, \partial g_0) : (M', \partial M') \to (X, \partial X)$$

with the following properties. For any $0 \leq j \leq k - 1$ the restriction of $(g_0, \partial g_0)$ on the transversal preimage of (X_j, X_{j+1}) is the s-triangulatin g_j of the manifold pair with boundaries (X_j, X_{j+1}). If a normal map $f : M \to X$ is an s-triangulation of the subfiltration

$$(17) \qquad\qquad\qquad X_{k-1} \subset \cdots \subset X_1 \subset X$$

and the restriction $f|_{M_k}$ is an s-triangulation of the pair (X_{k-1}, X_k), then by [5, Proposition 2.5], the map f is an s-triangulation of the filtartion (15) $\mathcal{X}$. Now we can apply this result $k - 1$ times starting from the subfiltration $X_1 \subset X_0$ of the filtration $X_2 \subset X_1 \subset X_0$ untill the subfiltration (17) of the filtartion (15). Corollary is proved. $\square$

REFERENCES

1. W. Browder – F. Quinn, *A surgery theory for G-manifolds and stratified spaces*, in Manifolds (1975), Univ. of Tokyo Press, 27–36.
2. M. M. Cohen, *A Course in Simply Homotopy Theory*, Springer-Verlag, New York, 1973.
3. S. Lopez de Medrano, *Involutions on Manifolds*, Springer-Verlag, Berlin–Heidelberg– New York, 1971.
4. Yu. V. Muranov – D. Repovš – F. Spaggiari, *Surgery on triples of manifolds*, Mat. Sbornik **8** (2003), 139–160; English transl. in Sbornik: Mathematics **194** (2003), 1251–1271.
5. Yurij V. Muranov – Dušan Repovš – Rolando Jimenez, *Surgery spectral sequence and manifolds with filtration*, Trudy MMO (in print) (2005).
6. A. A. Ranicki, *Exact Sequences in the Algebraic Theory of Surgery*, Math. Notes **26**, Princeton Univ. Press, Princeton, N. J., 1981.
7. C. T. C. Wall, *Surgery on Compact Manifolds*; Second Edition, A. A. Ranicki, Editor, Amer. Math. Soc., Providence, R. I., 1999.
8. S. Weinberger, *The Topological Classification of Stratified Spaces*, The University of Chicago Press, Chicago and London, 1994.

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